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CRITICAL SPACING FOR HEAVY QUARKONIUM DISSOCIATION

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Abstract

When a heavy quark and the corresponding antiquark are separated by more than $1.4 - 1.5$ fm, it becomes energetically favorable for a light quark-antiquark pair to be produced, leading to fragmentation into a pair of flavored mesons. The relation of this critical quark separation to other dimensional constants of the strong interactions (such as the pion decay constant, the QCD scale, and the light-quark constituent mass) is discussed.

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The dimensional nature of quantum chromodynamics is manifested in a number of different ways, all of which are plausibly equivalent to one another. For example: (1) The scale of low-energy pion-pion interactions is set by the ratios $p_i \cdot p_j / f_\pi^2$, where p_i are pion 4-momenta and $f_\pi = 132$ MeV is the pion decay constant. (2) A QCD scale $\Lambda_{\text{QCD}} = \mathcal{O}(200 - 400)$ MeV whose specific value depends on renormalization scheme is necessary in order to define the strong-coupling constant α_s at a suitable momentum scale. (3) The masses of u and d quarks as manifested in hadrons can be regarded as “constituent-quark” values m_q of order $300 - 400$ MeV. The scale of m_q , while not as precisely defined as f_π or Λ_{QCD} , must be related to these two quantities if, as expected, the limit of zero bare mass for u and d quarks makes sense.

In the present note we add a quantity to this list. There appears to be a critical separation between a heavy quark and a heavy antiquark for which it is energetically favorable to produce a pair of flavored mesons [as, for example, in $\Upsilon(4S) \rightarrow B\bar{B}$]. This feature was noted in early studies of the Υ system [1, 2]. However, the explicit magnitude of the interquark separation leading to heavy quarkonium dissociation was not presented. We find that it is of order $1.4 - 1.5$ fm. This number is of current interest for several reasons.

(1) Lattice gauge theories are approaching a stage where the dimensional quantities just mentioned (as well as others) can be related to one another. The study of light-quark production between color centers represented by a heavy quark and antiquark is becoming feasible [3] as one learns to cope with the “unquenched” version of QCD in which light quark-antiquark pairs are properly treated.

(2) The quarkonium systems to which the critical dissociation distance applies include $c\bar{c}$ and $b\bar{b}$ systems, for which there remain prospects for discovering a few more P-wave and D-wave states, and $b\bar{c}$ states, for which there are extensive theoretical studies of the spectroscopy [4] and a hint of the ground state [5]. The top quark is too heavy to have a quarkonium spectroscopy (since its decay width will be greater than the expected level spacing), but the critical-separation parameter should still apply to $t\bar{t}$ pairs or their products $t\bar{b}$, $\bar{t}b$, or $b\bar{b}$ [6].

(3) One might expect the critical separation of a pair of static color centers for dissociation into a pair of flavored mesons to be a fundamental parameter in theories [7] linking heavy-quark physics and chiral perturbation theory.

We assume that a flavor-independent potential $V(r)$ describes all bound states $Q\bar{Q}$ of a heavy quark Q and its antiquark as well as flavored mesons $Q\bar{q}$ and $\bar{Q}q$ involving the heavy quark Q and a light quark $q = u, d$. In the notation of Ref. [2], we define $\delta(m_Q) \equiv 2m(\text{lowest } Q\bar{q}) - 2m_Q$. This quantity should tend to a finite limit δ_∞ as $m_Q \rightarrow \infty$. The quantity $\delta_\infty/2$ is the same as the parameter $\bar{\Lambda}$ of heavy quark effective theory [8]. We seek the value of the critical threshold $Q\bar{Q}$ separation r_{th} for which $\delta_\infty = V(r_{\text{th}})$. A trivial modification permits the description of bound states such as $b\bar{c}$ involving unlike-mass quarks.

The value of $r_{\text{th}} \equiv V^{-1}(\delta_\infty)$ will be universal to the extent that (a) δ_∞ is well-defined (as expected in heavy quark effective theory), and (b) a flavor-independent potential $V(r)$ actually provides a good description of the $Q\bar{Q}$ interaction near flavor threshold. The parameters $\delta_\infty/2$ and the light-quark constituent mass m_q

are closely related; neither parameter can be chosen arbitrarily in a theory with given f_π or Λ_{QCD} . The assumption of a flavor-independent potential has been reasonably well borne out [9, 10] by comparison of $c\bar{c}$ and $b\bar{b}$ systems, and will be tested further in studies of $b\bar{c}$ states [4].

We calculate the critical separation using a simple phenomenological potential [11]. We then compare it with values obtained in a more model-independent manner, and discuss its relation to other dimensional constants in QCD.

A satisfactory interpolation between charmonium ($c\bar{c}$) and upsilon ($b\bar{b}$) states is provided by a potential of the form $V(r) = C \ln(r/r_0)$, with $C \simeq 0.72$ GeV and r_0 depending upon the specific choice of c and b quark masses [11]. The Schrödinger equation for the reduced radial wave function $u(r)$ of S-wave bound states is

$$-\frac{1}{2\mu} \frac{d^2 u}{dr^2} + C \ln\left(\frac{r}{r_0}\right) u = Eu \quad , \quad (1)$$

where μ is the reduced mass: $2\mu = m_Q$ for a $Q\bar{Q}$ bound state, and $E = M(Q\bar{Q}) - 2m_Q$. In terms of dimensionless variables $\epsilon \equiv E/C$ and $\rho \equiv r\sqrt{2\mu C}$, (1) becomes

$$-\frac{d^2 u}{d\rho^2} + \ln \rho u = [\epsilon + \ln(r_0\sqrt{2\mu C})]u \quad . \quad (2)$$

The lowest eigenvalue of this equation is $\epsilon + \ln(r_0\sqrt{2\mu C}) = 1.0443$, based on a numerical solution [12]. Consequently, one may eliminate the parameter r_0 in favor of the ground state mass, the parameter C , and the reduced mass. Recalling the definition of the radius r_{th} and setting $M_{\text{th}} - 2m_Q = V(r_{\text{th}})$, we finally have

$$[M_{\text{th}} - M(1S)]/C + 1.0443 = \ln(r_{\text{th}}\sqrt{2\mu C}) \quad . \quad (3)$$

We begin by neglecting spin-dependent effects in both $Q\bar{Q}$ and $Q\bar{q}$ systems, and reduced-mass effects in the $Q\bar{q}$ system. This approximation is most reliable for the Υ levels. Consequently, taking $M_{\text{th}} = 2M(B) = 10.558$ GeV, $M(1S) = M[\Upsilon(1S)] = 9.460$ GeV, $C = 0.72$ GeV, and a range of $2\mu = m_b$ between 4.5 and 5 GeV, we find the values of r_{th} shown as the dashed line in Fig. 1.

To account for the hyperfine splittings in the $Q\bar{Q}$ and $Q\bar{q}$ systems and the reduced-mass effect in the $Q\bar{q}$ system, we assume that the $\Upsilon(1^3S_1)$ level is 10 MeV above the spin-averaged $1S$ mass [13]. Furthermore, we apply the corrections [2]

$$\delta(m_Q)_{\text{hfs}} = \frac{m_b}{m_Q} \cdot \frac{3}{2}(m_{B^*} - m_B) = 69 \text{ MeV} \left(\frac{m_b}{m_Q} \right) \quad (4)$$

and

$$\delta(m_Q)_{\text{red. mass}} = -C \ln[\mu(m_Q)/\mu(m_b)] \simeq Cm_q \left(\frac{1}{m_Q} - \frac{1}{m_b} \right) \quad (5)$$

to estimate

$$\delta_\infty - \delta(m_b) \simeq 10 \text{ MeV} + 69 \text{ MeV} - 46 \text{ MeV} = 33 \text{ MeV} \quad , \quad (6)$$

where the first term is our estimate of the $\Upsilon(1S)$ hyperfine term, the second is the contribution of the hyperfine term in $2M(B)$, and the third is the reduced-mass effect, estimated for the logarithmic potential. One is thus assuming such a potential to hold not only for $Q\bar{Q}$ but also for $Q\bar{q}$ states. We have used a light-quark constituent mass [14] $m_q = 310$ MeV. With the above corrections, we now estimate the range of r_{th} shown as the solid line in Fig. 1. The corrected values of r_{th} range between about 7.6 and 7.2 GeV^{-1} (1.5 – 1.42 fm) for $4.5 \leq m_b \leq 5$ GeV.

For comparison, the dot-dashed line in Fig. 1 depicts the values of r_{th} obtained from an inverse-scattering construction of the interquark potential [9] using the Υ levels. These values have not been corrected for hyperfine or reduced-mass effects, so they should be compared with the uncorrected values obtained above. A power-law potential [15] fitting charmonium and Υ spectra with $(m_c, m_b) = (1.56, 4.96)$ GeV gives $r_{\text{th}} = 7.2$ GeV^{-1} (uncorrected) and 7.6 GeV^{-1} (corrected). The agreement of the various estimates is fairly good, indicating that model-dependent effects are unlikely to affect the estimate significantly. As shown in inverse-scattering [9] and explicit potential [10] calculations, the shape of any smooth potential which reproduces a given set of energy levels is fairly well specified for the range of energies corresponding to the known levels. Once a potential is required to reproduce the $\Upsilon(1S - 4S)$ levels and their leptonic widths, that potential's shape is specified between roughly $0.1 \text{ fm} = 0.5 \text{ GeV}^{-1}$ and the interquark separation corresponding to $V(r) \approx M[\Upsilon(4S)] - 2m_b$, which is just above flavor threshold.

Lattice gauge theories are able to estimate the distance at which a string of chromoelectric flux breaks by the deviation from a linear behavior of the field energy as a function of interquark separation. It is not clear that such a distance corresponds to flavor threshold, since discrete $b\bar{b}$ resonances (and indeed, series of light-quark resonances) persist well above flavor threshold. Nonetheless, our estimate of r_{th} is in accord with an upper bound of $1.9 \pm 0.2 \pm 0.2$ fm [16] (quoted in Ref. [3] as 1.7 fm on the basis of a calculation in Ref. [17]) for the breaking of a QCD string obtained using a quenched approximation in lattice gauge theory. In a quenched lattice of size $(1.5 \text{ fm})^3$, no breaking of a QCD string has been observed; the linear behavior persists out to the maximum accessible interquark separation. It would be interesting to see whether at slightly larger distances a linear behavior could actually *coexist* with production of light quark-antiquark pairs.

How might the critical spacing parameter r_{th} be related to other dimensionful quantities in QCD? It is clearly related to the constituent-quark mass since a certain amount of chromoelectric energy is required to create the $q\bar{q}$ pair. In turn, the constituent-quark mass scale is set [14, 18] by the need to agree with such quantities as m_ρ and m_p , whose values are related to the QCD scale Λ_{QCD} . It has been argued [19] (cf., however, Refs. [20]) that the mass scale of resonances like the ρ meson can be regarded as a number of order $2\pi f_\pi$ in a chirally symmetric theory involving massless pions. In any event a direct relation between f_π and Λ_{QCD} in a theory of massless pions is highly likely.

Further dimensionful quantities in the strong interactions which might bear a relation to those mentioned include the universal string tension describing the long-distance interquark interaction $V(r) \simeq kr$, $k \simeq 0.18 \text{ GeV}^2$, and the universal

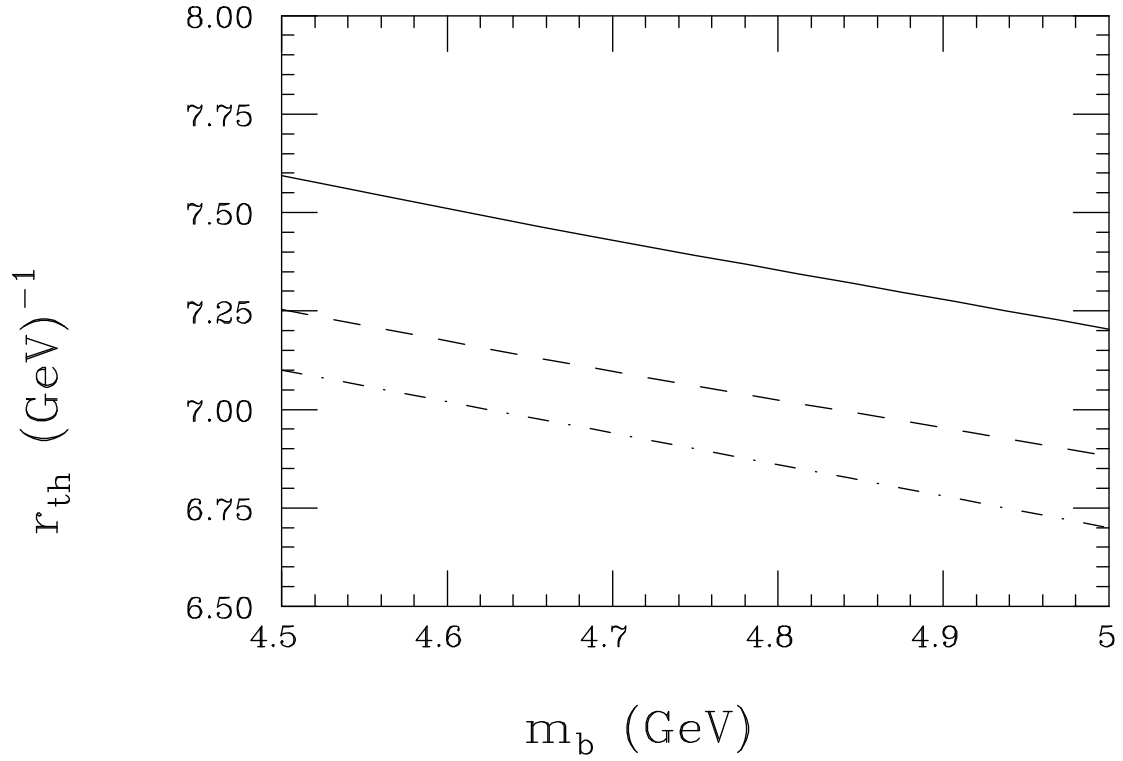


Figure 1: Dependence of threshold parameter r_{th} , as estimated from Υ levels, on assumed value m_b of bottom quark mass. Dashed line: uncorrected values assuming logarithmic potential. Solid line: values assuming logarithmic potential corrected for hyperfine and reduced-mass effects. Dash-dotted line: uncorrected values obtained from an inverse-scattering approach.

slope α' of Regge trajectories for light-quark systems, $\alpha' = 1/(2\pi k) \simeq 0.88 \text{ GeV}^{-2}$ [21]. The production of light quark-antiquark pairs in a linear potential has been considered some time ago [22].

To conclude, we have argued that once a heavy color triplet and antitriplet become separated by more than $1.4 - 1.5 \text{ fm}$, the chromoelectric flux lines joining them contain sufficient energy to produce a light quark-antiquark pair, leading to the decay of the heavy quarkonium system into a pair of flavored mesons. It would be interesting to see whether current lattice gauge and chiral theories of non-perturbative QCD could relate this quantity to others which set the QCD scale.

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